Stimulated Enhancement of Cross-Section by a Bose-Einstein Condensate

J.J. Hope* and C.M. Savage
Department of Physics and Theoretical Physics, The Australian National University,
Australian Capital Territory 0200, Australia.
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This paper examines the feasibility of constructing an experiment which detects the atomic stimulation of a photon emission process. A beam of atoms (bosons) in an excited state is passed over an atomic trap which traps the atoms when they are in their internal ground state. When the trap contains a Bose-Einstein condensate, the cross-section for absorption of the atomic beam is increased. We examine a particular model in which this *atom*-stimulation is observable, and is also characterised by the emission of photons in a narrow cone in the direction of the atomic beam.

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A large number of atoms in a single quantum mechanical (internal and external) state has been produced in atomic traps in the recent experiments which have produced a Bose-Einstein condensate (BEC) [1–3]. This high degree of quantum degeneracy may be used as an atomic source for atom optics experiments [4–7], or to demonstrate new effects which depend entirely on the quantum statistics. The stimulation of a transition by a BEC has not been observed. We propose a method by which the emission of photons may be observed to be stimulated by the presence of a BEC.

The emission of a photon from an atom in an excited state can be stimulated by a large number of photons in the destination mode, as in a laser. If the atoms involved are bosons, then the same emission can also be stimulated by the presence of many atoms in the destination *atomic* mode. Any transition rate between two states is enhanced by a factor of $(N+1)$ where N is the number of bosons occupying the final state.

An atom in a metastable excited state can pass through an atomic trap which only traps the ground (internal) state. If the trap contains a condensate of N atoms, then there will be an enhancement of the fraction of the spontaneous emission which goes into the ground (trap) state, and therefore an enhancement of the overall emission rate. A schematic of this process is shown in figure 1. If we let γ be the free space spontaneous emission rate and f be the fraction of the atoms which will spontaneously emit into the ground (trap) state, then the total emission rate γ_{tot} is the sum of the spontaneous emission rate into the non-ground states and the spontaneous plus the stimulated emission rates into the ground state

$$\gamma_{tot} = \gamma(1-f) + (N+1)\gamma f = \gamma + \gamma N f. \tag{1}$$

The rightmost term of this equation will be called the *stimulated* emission rate, referring to the photon emission which is stimulated by a highly populated *atomic* mode. It is clear from this equation that for a BEC with a sufficiently large number of atoms, the stimulated emission rate will dominate the spontaneous emission rate, and that the fraction f will determine the critical size of such a BEC. In this letter, we will calculate the ratio of stimulated emission to spontaneous emission for a particular model, and show that for experimentally realisable parameters it is possible for the stimulated emission rate to become much larger than the spontaneous emission rate, significantly increasing the overall emission rate. In this particular model, the stimulated emission will occur in a narrow cone around the direction of motion of the excited atoms.

We denote the center of mass wavefunction of an excited (internal) state atom in the beam by $\Psi(\mathbf{x},t)$. If we also denote the single particle ground (internal and trap) state by $\Phi(\mathbf{x})$, and the momentum kick produced by the photon emission by $\hbar\mathbf{k}$, then the fraction f of atoms which will spontaneously emit into the ground (internal and trap) state is given by the overlap integral

$$f(t) = \int_\Omega \frac{d^2\mathbf{k}}{4\pi k_o^2} \left\| \int \mathbf{dx}\Phi^*(\mathbf{x})\Psi(\mathbf{x},t)e^{i\mathbf{k}.\mathbf{x}} \right\|^2, \tag{2}$$

where $\hbar k_o$ is the absolute momentum kick given by the emission of the photon and $\int_\Omega d^2\mathbf{k}$ denotes the integral in k-space of all possible photon directions.

We now assume that the excited state wavefunction will be travelling with some narrow momentum distribution centred around $(\hbar k_o, 0, 0)$, so that there will be a strong resonance in the integral around $\mathbf{k} = (-k_o, 0, 0)$. This means that we can ignore all contributions to the k-space integral except those very close to this point, and the photons will emit in a very narrow cone in the direction of travel of the excited atom. This means that the transverse (y,z) integrals will be trivial, and we will approximate them to be unity. This approximation is valid if the transverse wavefunction of the excited atom has the same shape as the transverse wavefunction of the destination state. The overlap integral is then approximately one dimensional

$$f(t) = \int_{-k_o}^{k_o} \frac{dk}{2k_o} \left\| \int_{-\infty}^{\infty} dx\Phi^*(x)\Psi(x,t)e^{ik.x} \right\|^2. \tag{3}$$

We now consider a specific case to calculate this overlap integral, and determine the feasibility of producing

an experiment which will measure *atom*-stimulated emission. We consider the case where both wavefunctions have a Gaussian form, and the excited state wavefunction $\Psi(x,t)$ is shifted to a centre of momentum of $\hbar k_o$

$$\Phi(x) = (\frac{1}{2\pi l_1^2})^{\frac{1}{4}} \exp\left[-\frac{x^2}{4l_1^2}\right], \tag{4}$$

$$\Psi(x,t) = (\frac{1}{2\pi l_2^2 s(t)})^{\frac{1}{4}} \exp\left[i(k_o x - \omega_o t)\right] \times \exp\left[-(x - v_o t)^2/(4l_2^2 s(t))\right], \tag{5}$$

where l_1 and l_2 are the sizes of the Gaussian wavefunctions of the ground state and the excited state wavefunctions respectively, $\omega_o = \hbar k_o^2/(2M)$, $v_o = \hbar k_o/M$, $s(t) = 1 + i\hbar t/(2Ml_2^2)$ is a measure of the spreading of the beam, and M is the mass of the atom. We have assumed here that the source atomic beam and the ground state of the trap have been perfectly aligned, and we have set the zero of time at the moment when they are exactly coincident.

These wavefunctions are chosen to be Gaussian because that is the form of the ground state of a harmonic trap. A possible method of producing the excited state atoms in this form would be to produce a BEC in a second trap, and then use a laser pulse to transfer the atoms to a metastable state via a Raman transition. A Raman transition would leave the atoms in a metastable internal state with an overall momentum kick $\hbar k_o$ due to the difference in photon momenta. This method would be appropriate when the kick was required to be small. We will show that the largest number of atoms are drawn into the BEC when a *small* kick is given to the atoms. This type of atom source has several natural advantages for this experiment. Firstly, the BEC is a very cold atom source and will allow all of the atoms to be in the energy range which can be trapped by the target BEC. Secondly, using copropagating lasers to produce the Raman transition will provide naturally the required resonance between the motional state of the atom (Gaussian wavepacket travelling with average momentum $\hbar k_o$) and the kick ($\hbar k_o$) due to the photon which may be emitted.

We now make the assumption that the excited state wavepacket does not spread while it overlaps the target wavefunction $\Phi(x)$ ($s(t) = 1$), or precisely that $\Psi(x,t) = \Psi(x - v_o t, 0)$. This approximation is made for calculational purposes, and is true for timescales $\tau \ll 2Ml_2^2/\hbar$. This implies the condition

$$k_o \gg \sqrt{l_1^2 + l_2^2}/(2l_2^2). \tag{6}$$

We now calculate the fraction $f(t)$ of atoms spontaneously emitting into the ground state under these assumptions. From Eqs. (3,4,5) we obtain the result:

$$f(t) = \frac{\sqrt{\pi}}{2k_o\sqrt{2(l_1^2 + l_2^2)}} \operatorname{Erf}\left[\frac{2\sqrt{2}k_o l_1 l_2}{\sqrt{l_1^2 + l_2^2}}\right] \times \exp\left[-(v_o t)^2/(2(l_1^2 + l_2^2))\right]. \tag{7}$$

In general, the excited atoms will have a very weak spontaneous emission rate, which is due to the low frequency of the emitted photon. This means that we can ignore depletion of the excited state atoms when calculating the number of spontaneous emission events. We also ignore depletion when calculating the total number of stimulated emissions. Including depletion is a straightforward extension, and can only become important after the stimulated emission already has increased the total number of emission events by a large factor.

In an experiment, it is likely that the atoms will travel some distance D between the source and the detectors which is large compared to the size of the trap or the atomic beam, l_1, l_2. The atoms will be emitting spontaneously for the entire distance, but the stimulated emission will be insignificant unless they are overlapping with the trap ground state. Integrating each of the two terms of the right hand side of equation (1) with respect to time will allow us to calculate the total number of stimulated emission events and the total number of spontaneous emission events. We denote the ratio of these by R, which is given by

$$R = \frac{N\pi}{2Dk_o} \operatorname{Erf}\left[\frac{2\sqrt{2}k_o l_1 l_2}{\sqrt{l_1^2 + l_2^2}}\right]. \tag{8}$$

The dependence of this ratio on N was expected, as the stimulated emission rate is directly proportional to the number of atoms in the BEC so the total number of stimulated emissions will be proportional to N. The total number of spontaneous emissions is proportional to the time of travel of the excited atomic cloud. This in turn is proportional to the distance D, so we expected the ratio R to be inversely related to D. The dependance on k_o is a consequence of our particular model.

Figure 2 plots the value of R as a function of k_o, and shows that smaller values of k_o produce a stronger stimulated signal. The other restriction on k_o is provided by the calculational requirement that the excited state wavepacket doesn't spread over time. Over the region of the trap, this will be true if inequality (6) is satisfied.

Substituting some realistic numbers into this equation will show the feasibility of conducting such an experiment. We choose the number of atoms in the condensate to be $N = 5 \times 10^5$, which has been achieved already in experiments [3]. The size of both the incident wavepacket and the target ground state are $l_1 = l_2 = 30\mu$m, which is a couple of times larger than current experiments. The total distance travelled by the excited atoms is chosen to be $D = 2$cm, but this should be as small as possible in a well designed experiment, and in the theoretical limit can be as small as the traps themselves. For a value of $k_o = 4 \times 10^6\text{m}^{-1}$, the excited state wavepacket increases in size by about half a percent while passing over the

ground state of the trap. These parameters give a ratio of $R = 10$ times more stimulated emission events than spontaneous emission events. If the spontaneous emission is very weak but still measurable, then there will be a significant increase in the emission rate when the BEC is present in the trap.

There are two possible methods for detecting the emission of photons. The first is to directly measure the proportion of excited state atoms which manage to pass through the trap. For weak spontaneous emission but strong stimulated emission, this will change as the BEC absorbs the atomic beam. Another method of detection which can be carried out simultaneously is to detect the photons which are emitted. Spontaneously emitted photons will come from the condensate at a random direction, whereas the radiation due to the stimulated process will emit in a narrow cone in the direction of travel of the excited state atoms. For the parameters used in the previous calculation, the maximum deviation from the center of the cone would be of the order of 9^o, corresponding to a solid angle of $0.006 \times 4\pi$. This angle is smaller for larger values of k_o, l_1 and l_2. The background spontaneous emission rate over this small solid angle will therefore be correspondingly reduced, increasing the signal to noise ratio by several orders of magnitude.

If we consider the effect of collecting the light over a small solid angle, we can arrange it so that nearly all of the stimulated photons are collected while only a small proportion of the spontaneously emitted photons are collected. We estimate the size of this relative increase by estimating the size of the k-space resonance and dividing it by the total area of the k-sphere. The k-space resonance of the stimulated signal for small solid angles can be derived from equations (3,4,5), which show that it has a width of approximately $\sqrt{1/l_1^2 + 1/l_2^2}$. This gives us an approximation to the ratio R_p of photons produced by stimulated and spontaneous emission within the reduced solid angle

$$R_p \approx \frac{N\pi}{D\sqrt{1/l_1^2 + 1/l_2^2}} \,\mathrm{Erf}\left[\frac{2\sqrt{2}k_o l_1 l_2}{\sqrt{l_1^2 + l_2^2}}\right]. \tag{9}$$

We can see that for this detection scheme, if we have k_o large enough to ensure a small solid angle (required by our original approximations), then the error function will be unity, and the result will be independent of k_o, depending only on the size of the trapped state and the excited state wavefunction. This detection scheme may allow the use of a single photon transition to produce the source beam while still measuring the effect of atom-stimulation, but for large k_o the depletion of the excited state beam will be insignificant.

Gravity has not been considered in this model, but will be important over the timescales involved with this experiment. It may be possible to utilise gravity in a particular experimental arrangement, but it may be necessary to balance the gravitational force. This might be achieved with a far-detuned light force [8] or an atom waveguide such as a hollow optical fiber [9–11].

Superradiance will not enhance the background spontaneous emission rate provided that the *source* of atoms is sufficiently dilute that on average, the atoms are much further apart than one optical wavelength. This condition can be easily satisfied, as there is no fundamental theoretical reason to use a high density source, and the stimulated effect has been shown in equation (8) to be largest when the wavepacket of the atomic *source* is made as physically large as possible. We also assume that the effect of collisions between the excited state atoms and the ground state atoms is negligible. This is true provided the *target* BEC is sufficiently dilute. This is a restriction on the density of the target BEC, but we have also shown that the stimulated emission rate is maximal when the *target* ground state wavefunction is physically as large as possible.

Reabsorption of the emitted photons by the condensate will reduce the spontaneous and stimulated emission signals by an equal proportion [12]. This effect will not alter the signal to noise ratio, but it would tend to heat the target BEC. We have neglected the effects of reabsorption of the beam.

This paper has shown that it should be feasible to produce an experiment which directly measures a process which is stimulated by a large number of bosonic atoms in a single quantum mechanical state. If these experiments can be produced such that the stimulated emission completely dominates the spontaneous emission, and a significant number of atoms are transferred into the BEC, then this may be a method of increasing the number of atoms in a BEC beyond that made possible by evaporative cooling.

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* Email address: Joseph.Hope@anu.edu.au

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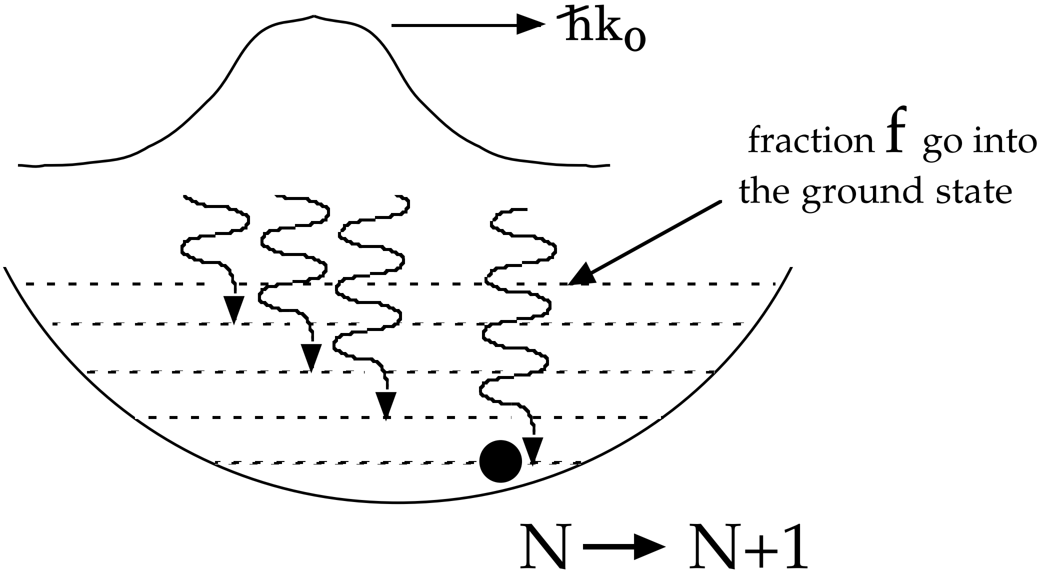
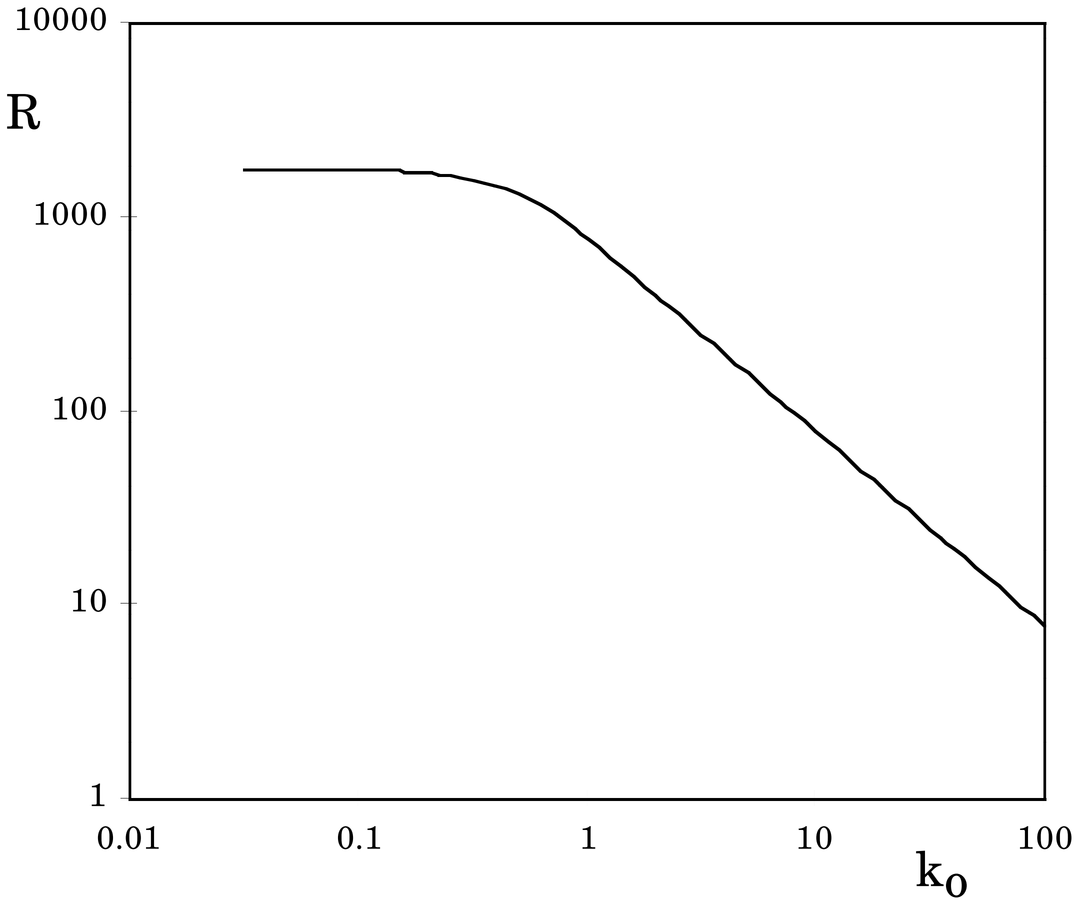
FIG. 1. Schematic of the modelled system. The top wavepacket shows the beam of atoms in the excited state. The other smooth solid line shows the trap potential for the ground electronic state of the atom. The dashed lines show the energy levels in the trap, with a large population in the the ground trap state. The waved lines show transitions into various trap states.

FIG. 2. Ratio (R) of stimulated emission to spontaneous emission. The chosen parameters are $N = 5 \times 10^5$, $D = 10$cm and k_o is in units of $(l_1^2 + l_2^2)^{1/2}/(l_1 l_2)$.



$\hbar k_o$
fraction f go into the ground state
$N \rightarrow N+1$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